MIDTERM EXAM II

Last name: Name:

BUID:

Please do all of your work in this exam booklet and make sure that you cross any work that we should ignore when we grade. Books and extra papers are not permitted. If you have a question about a problem, pease ask. Remember: answers that are written logically and clearly will receive higher scores. Calculators are not allowed.

Problem n^o	**Possible points**	**Score**
1	10	
2	10	
3	15	
4	10	
5	20	
6	15	
7	20	
Total:	100	
Bonus:	10	

Problem 1. Find the derivatives of the following functions:

- $f(x) = e^{2x} \sin(x)$
- $g(x) = \sqrt[3]{x^3 - 1/x}$
- $p(x) = 3x^4 - 2x^3 + \dfrac{2}{x^3}$
- $h(x) = \sin(x) \ln(\cos(x))$
- $m(x) = \dfrac{\ln(1 + x)}{x^3}$

Problem 2. Decide if the following functions are possible or impossible. For each possible case, sketch its graph.

- A function with 3 vertical asymptotes.

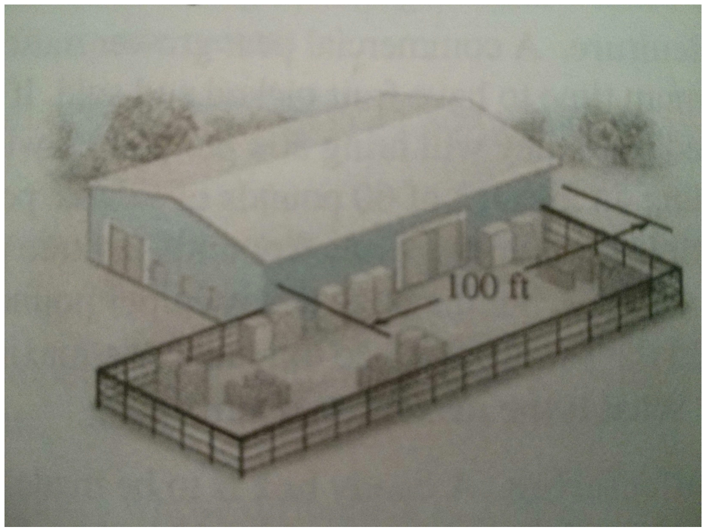
- A function with 3 horizontal asymptotes.

- A function with 2 local maxima and 1 local minimum.

- A function with 2 local maxima and no local minima.

- A function with 3 inflection points.

Problem 3. The owner of a retail lumber store wants to construct a fence to enclose an outdoor storage area adjacent to the store, using all of the store as part of one side of the area. Find the dimensions (length of both sides) that will enclose the largest area if 400 feet of fencing material are used.



BONUS: Try to repeat the same but with 240 feet of fencing instead. What goes wrong? What is the answer in this case?

Problem 4. Sketch a function f satisfying the following conditions:

- $\text{Dom} f = [0, \infty)$
- $f(0) = 0$
- $\lim_{x \to \infty} f(x) = 0$
- $f' > 0$ in $(2, 5)$ and $f' < 0$ in $(0, 2)$ and $(5, \infty)$
- $f'' > 0$ in $(0, 3)$ and $(6, \infty)$ and $f'' < 0$ in $(3, 6)$

BONUS: Think of a realize process/situation that could be modeled by a function producing the above graph.

Problem 5. A fish is reeled in at a rate of 2 foot per second from a point 6 feet above the water. At the instant there is a total of 10 feet of line out.

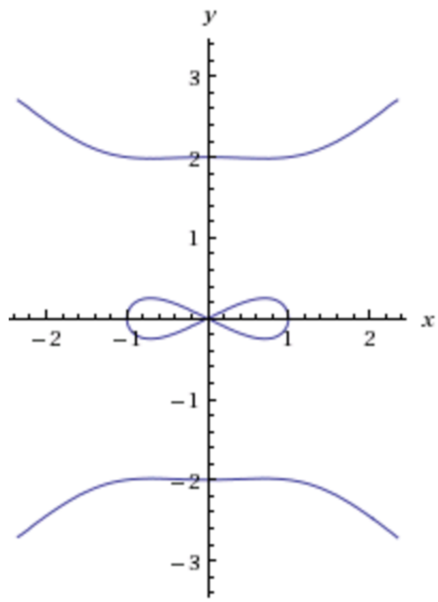
- Draw a picture of the situation labeling the variables, their rates and known values.

- How far from the pier is the fish?

- At what rate is the distance between the fish and the pier changing?

- At that same moment, what are the sine, cosine and tangent of the angle between the line and the water? At what rate is this angle changing?

Problem 6. The curve $y^4 - 4y^2 - x^4 + x^2 = 0$ is known as the Devil's curve, and has the following graph.



- Check that the point $(1, 2)$ belongs to the curve and use implicit differentiation to find the equation of the tangent line at that point.

- Draw the tangent line in the picture above to check your answer.

Problem 7. Use the guidelines introduced in class to produce a precise graph of the function
$$f(x) = \frac{x}{2 - x^2}.$$

- $\mathrm{Dom}(f) = \mathbb{R} \setminus \{-\sqrt{2}, \sqrt{2}\}$.
- Odd function: $f(-x) = -f(x)$.
- $f'(x) = \dfrac{2 + x^2}{(2 - x^2)^2}$. $f'(x) = 0 \iff x^2 + 2 = 0 \iff x = \pm\sqrt{-2}$ DNE
- $f''(x) = \dfrac{2x(x^2 + 6)}{(2 - x^2)^3}$. $f''(x) = 0 \iff x = 0$ or $x^2 + 6 = 0$. So only $x = 0$.
- $f' > 0$ always, so f always increasing.
- $f'' > 0$ in $(-\infty, -\sqrt{2})$ and $(0, \sqrt{2})$. $f'' < 0$ in $(-\sqrt{2}, 0)$ and $(\sqrt{2}, \infty)$.
- V.A: $x = \sqrt{2}$ and $x = -\sqrt{2}$.
- H.A: $\lim_{x\to\pm\infty} f(x) = 0$, so $y = 0$.

With this info the graph is: