QUIZ I (Solutions)

Problem 1. Given the functions $f(x) = x^2 - 1$ and $g(x) = \dfrac{2}{x-1}$, find expressions for $g^{-1}(x)$, $(f \circ g)(x)$ and $(g \circ f)(x)$.

- For finding the inverse, change $g(x)$ by x and x by $g^{-1}(x)$.

$$x = \frac{2}{g^{-1}(x) - 1} \Longrightarrow x(g^{-1}(x) - 1) = 2 \Longrightarrow g^{-1}(x) = \frac{2}{x} + 1.$$

- $f \circ g$:

$$(f \circ g)(x) = f(g(x)) = \left(\frac{2}{x-1}\right)^2 - 1 = \frac{4}{(x-1)^2} - \frac{(x-1)^2}{(x-1)^2} = \frac{-x^2 + 2x + 3}{(x-1)^2}.$$

- $g \circ f$:

$$(g \circ f)(x) = g(f(x)) = \frac{2}{(x^2-1) - 1} = \frac{2}{x^2 - 2}.$$

Problem 2. Suppose you are given the following table of values corresponding to a certain function f:

x	0.9	0.99	0.999	1.1	1.01	1.001	1.0001
$f(x)$	3.0435	3.0101	3.0012	3.1245	4.3421	-1.983	10.9532

What would you say about the value of the following limits?

- $\lim\limits_{x \to 1^-} f(x) = 3$ (Strong evidence from table)
- $\lim\limits_{x \to 1^+} f(x) =$ DNE (Not a clear pattern, if your answer was 11 also good but worse)
- $\lim\limits_{x \to 1} f(x) =$ DNE (One sided limits don't match)

Can you assure your answers are the true limits, or just the more likely?

We can't assure it, the table only suggests what is more likely.

Problem 3. Use the graph of the function h in the figure to find the following values or state that they do not exist. If a limit does not exist, explain why.

From the graph:

a. $h(-2) = -1$

b. $\lim_{x \to -2^-} h(x) = 2$

c. $\lim_{x \to -2^+} h(x) = -1$

d. $\lim_{x \to -2} h(x) = \text{DNE}$

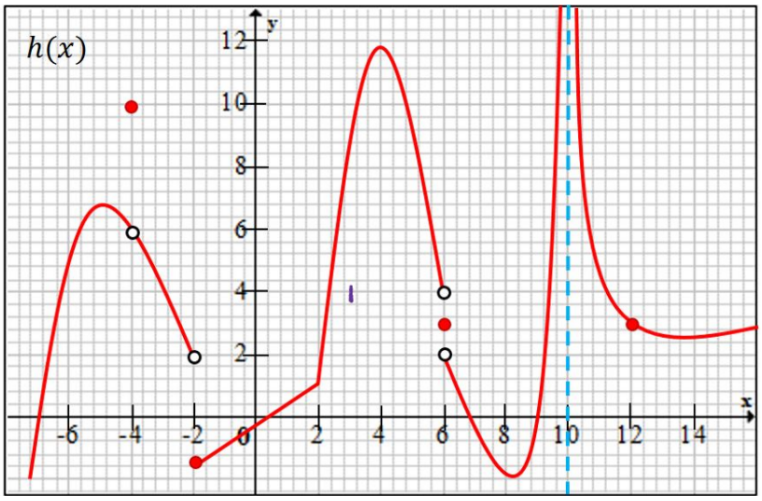
e. $h(2) = 1$

f. $\lim_{x \to -4} h(x) = 6$

g. $\lim_{x \to 6^+} h(x) = 2$

h. $\lim_{x \to 6} h(x) = \text{DNE}$

i. $h(6) = 3$



In points **d.** and **h.** the overall limits don't exist because the one-sided limits do not coincide.

Since the graph is approximate, the answers have been considered wrong only if they differ significantly.