QUIZ III

Problem 1. Compute the following limits and antiderivatives. For the antiderivatives, **check your answer** by differentiating. (Hint: Remember the constant + C !!!)

- $\lim_{x\to 0} \frac{\sin(x)}{x} = \frac{0}{0}$ by direct substitution, so we can use l'Hôpital's Rule.

$$\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} \frac{\cos(x)}{1} = \frac{1}{1} = \boxed{1}.$$

- $\lim_{x\to 0} \frac{e^{3x} - 3x - 1}{x^2} = \frac{0}{0}$, so again we can use l'Hôpital's Rule (actually twice).

$$\lim_{x\to 0} \frac{e^{3x} - 3x - 1}{x^2} = \lim_{x\to 0} \frac{3e^{3x} - 3}{2x} = \lim_{x\to 0} \frac{9e^{3x}}{2} = \boxed{\frac{9}{2}}.$$

- $\int 2\sin(x)dx = 2\int \sin(x)dx = 2(-\cos(x)) + C = \boxed{-2\cos(x) + C}.$

  Note that $\cos^2(x)$ can't be the antiderivative because its derivative is

$$\frac{d}{dx}\left(\cos^2(x)\right) = -2\cos(x)\sin(x) \neq 2\sin(x).$$

- $\int (x^3 - 2x + 1)dx = \boxed{\frac{x^4}{4} - x^2 + x + C}$

- $\int \sec^2(x)dx = \boxed{\tan(x) + C}$. Recall that $\frac{d}{dx}\tan(x) = \sec^2(x)$.

- $\int 2e^{3x}dx = \boxed{\frac{2e^{3x}}{3} + C}.$

Problem 2. Use $f(x) = x^3$ and linear approximation at the point $x = 2$ to approximate the value of 2.1^3 (3 pts).

The equation for the approximation at the point 2 (same as tangent line at that point) is:

$$L(x) = f'(2)(x-2) + f(2).$$

We have that $f(2) = 2^3 = 8$, and that $f'(2) = 3 \cdot 2^2 = 12$, so

$$\boxed{L(x) = 12(x-2) + 8}.$$

Now we use it to find $2.1^3 = f(2.1) \simeq L(2.1) = 12(2.1-2)+8)12\cdot 0.1+8 = 1.2+8 = \boxed{9.2}$ The actual value is 9.261 (you did not have to compute it, but is was doable).