Working With Taylor Series

This exercises are meant to exemplify the utilities of using Taylor series. Besides the obvious usage in approximating functions, they can be used in a variety of situations, including computing limits, integrals and even differential equations.

Limits

Use the Taylor of basic functions to compute the following limits:

- $\displaystyle\lim_{x \to 0} \frac{x^2}{\frac{1}{1-x} - e^x}$

- $\displaystyle\lim_{x \to 0} \frac{\arctan(6x) - \tan(6x)}{x^3}$

Differential equations

Consider the following dfferential equation

$$y'(x) - y(x) = 0.$$

We already know that the function $y(x) = e^x + C$ is its general solution, but now we will solve it using another method.

$$y(x) = \sum_{k=0}^{\infty} c_k x^k.$$

- Differentiate the above expression to find one for $y'(x)$.

- Find the relations between the coefficients.