QUIZ I (Solutions)

Problem 1. The velocity of a jogger traveling along a straight road is given in feet per second by $v(t) = t(t-2)(t-4)$ for $0 \leq t \leq 3$ where t represents time in seconds.

- Write an expression for the acceleration.

$$a(t) = dv(t)/dt = d(t^3 - 6t^2 + 8t)/dt = 3t^2 - 12t + 8.$$

- What is the displacement of the jogger during the time interval $0 \leq t \leq 3$?

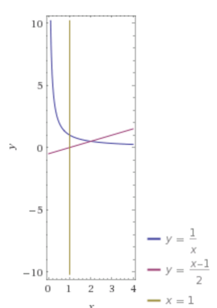
$$\int_0^3 v(t)dt = \left[t^4/4 - 2t^3 + 4t^2\right]_0^3 = 81/4 - 54 + 36 = 9/4.$$

- What is the total distance traveled by the jogger during the time interval $0 \leq t \leq 3$? Since the function $v(t)$ is negative from 2 to 3, we separate the integral into two parts.

$$\int_0^2 v(t)dt + \int_2^3 (-v(t))dt = 4 + 7/4 = 34/4.$$

Problem 2.

- Sketch the region enclosed by the curves $y = 1/x$, $y = (x-1)/2$ and $x = 1$.



- Compute the exact area of this region. First we need to fin the intersection between the two functions, which occurs at $x = 2$:

$$1/x = (x-1)/2 \iff x^2 - x - 2 = 0 \iff x = -1, x = 2.$$

Therefore the area is

$$\int_1^2 (1/x - (x-1)/2)dx = \left[\ln(x) - x^2/4 + x/2\right] = \ln(2) - \ln(1) - 1/4 = \ln(2) - 1/4.$$

Problem 3. The base of a solid is the region bounded by $y = 0$, $y = \sqrt{x}$ and $x = 1$. Cross-sections of the solid perpendicular to the x axis are squares. Sketch the solid and an exemplifying cross section. Write and expression for the area of the cross-sections depending on x and write an expression for the volume of the solid (no need to compute).

I will draw the picture in class. The area is the square of the function $A = y^2 = (\sqrt{x}^2) = x$. Thus the volume is

$$V = \int_0^1 A(x) = \int_0^1 x.$$