QUIZ II

Solve:

- $\int x \ln(x) dx$

  Use integration by parts: $u = \ln(x)$, $dv = x$. Then find $du = 1/x$ and $v = x^2/2$ and apply formula:

$$\frac{x^2 \ln(x)}{2} - \int \frac{x^2}{2x} dx = \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C$$

- $\int \frac{x + 4x^2 + x^2 \cos(x)}{x^2} dx$

  Just simplify and integrate:

$$\int \left(\frac{1}{x} + 4 + \cos(x) \right) dx = \ln(|x|) + 4x + \sin(x) + C$$

- $\int \cos^3(x) \sin^2(x) dx$

  Use substitution $\cos^2(x) = 1 - \sin^2(x)$ and then u-substitution $u = \sin(x), du = \cos(x)dx$:

$$\int \cos(x)(\sin^2(x) - \sin^4(x))dx = \int (u^2 - u^4)du = \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C$$

- $\int \frac{1}{1+x^2} dx = \arctan(x) + C$

- $\int \frac{1+x}{x^2 - x} dx$

  Use partial fraction decomposition: $x^2 - x = x(x-1)$.

$$\int \frac{2}{x-1} - \frac{1}{x} dx = [2\ln|x-1| - \ln|x|]_2^3 = 2\ln 2 - \ln 3 - 2\ln 1 + \ln 2 = \ln(8/3)$$

- $\int_0^2 \sqrt{4-x^2}dx$

  Use substitution $x = 2\sin(u)$, $dx = 2\cos(u)du$ and half angle formula.

$$\int \sqrt{4-4\sin^2(u)}2\cos(u)du = 4\int \cos^2(u)du = 4\int \frac{1+\cos(2u)}{2}du$$

$$= 2u + \sin(2u) + C = 2u + 2\sin(u)\cos(u) + C = 2\arcsin(x/2) + x\sqrt{1-\frac{x^2}{4}} + C$$

  Now plug in limits of integration:

$$\int_0^2 \sqrt{4-4x^2}dx = 2\arcsin(1) + 0 - 2\arcsin(0) + 0 = 2\pi/2 = \pi$$

  Alternatively you could just say it is one fourth of the area of the circle of radius 2!!!

- $\int x^2\sqrt{1-x^2}dx$

  Use trigonometric substitution $x = \sin(\theta)$, $dx = \cos(\theta)d\theta$ and then u-substitution $u = \sin(\theta)$.

$$\int \sin^2(\theta)\sqrt{1-\sin^2(\theta)}\cos(\theta)d\theta = \int \sin^2(\theta)\sqrt{\cos^2(\theta)}\cos(\theta)d\theta = \int \sin^2(\theta)\cos^2(\theta)d\theta$$

$$= \int \frac{1-\cos(2\theta)}{2}\frac{1+\cos(2\theta)}{2}d\theta = \int \frac{1-\cos^2(2\theta)}{4}d\theta = \int \frac{1}{4} - \frac{1+\cos(4\theta)}{8}d\theta$$

$$= \frac{\theta}{8} - \frac{\sin(4\theta)}{32} = \frac{\theta}{8} - \frac{\sin(2\theta)\cos(2\theta)}{16} = \frac{\theta}{8} - \frac{\sin(\theta)\cos(\theta)(\cos^2(\theta)-\sin^2(\theta)}{8}($$

$$= \frac{\arcsin(x) + \sqrt{1-x^2}(2x^3-x)}{8} + C$$