QUIZ III

Determine if the following series converge or diverge. Find the sum when possible.

- $\displaystyle\sum_{k=1}^{\infty} \frac{k^3+4k^2+12}{3k^4-2k+1}$

  Limit comparison test with $1/k$.

  $$\lim_{k\to\infty} \frac{\frac{k^3+4k^2+12}{3k^4-2k+1}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{k^4+4k^3+12k}{3k^4-2k+1} = \frac{1}{3}.$$

  Since it is a nonzero number, it has the same convergence type as $\sum 1/k$, and therefore it diverges.

- $\displaystyle\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\frac{3}{32}+\dots$

  Rewrite it explicitly as a geometric series.

  $$\sum_{k=1}^{\infty} \frac{3}{2}\left(\frac{1}{2}\right)^k = \frac{3}{2}\frac{1/2}{(1-1/2)} = \frac{3}{2}$$

  In particular it converges because the ratio $1/2 < 1$. .

- $\displaystyle\sum_{k=1}^{\infty} \frac{10^k}{k!}$

  Use ratio test:

  $$\lim_{k\to\infty} \frac{\frac{10^{k+1}}{(k+1)!}}{\frac{10^k}{k!}} = \lim_{k\to\infty} \frac{10^{k+1}}{10^k}\frac{k!}{(k+1)!} = \lim_{k\to\infty} \frac{10}{k+1} = 0.$$

  Therefore it converges.

- $\displaystyle\sum_{k=0}^{\infty} (-1)^k \ln\left(1+\frac{1}{k}\right) dx$

  It is an alternating series. Since

  $$\lim_{k\to\infty} \ln\left(1+\frac{1}{k}\right) = \ln(1) = 0,$$

  by the alternating series test in converges.

- $\displaystyle\sum_{k=2}^{\infty} \frac{1}{2k} - \frac{1}{2k+2}$

  Rewrite it as a telescoping series

$$\frac{1}{2}\sum_{k=2}^{\infty} \frac{1}{k} - \frac{1}{k+1} = \frac{1}{4}$$

  Note that the series starts at $k = 2$.

- $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k+1}{4-2k}\right)^{2k}$

  Use the root test

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{3k+1}{4-2k}\right)^{2k}} = \lim_{k\to\infty} \left(\frac{3k+1}{4-2k}\right)^{2} = \left(\frac{3}{-2}\right)^{2} = 9/4 > 1.$$

  It diverges.