Below are the questions corresponding to the second midterm exam. Please read the following instructions before starting:

- The exam must be submitted on Gradescope tomorrow, Friday 6/17 by **11:59AM Boston Time**. You may upload a scan or a picture of you documents. Late submissions won't be accepted. Please indicate which pages correspond to which question when uploading.
- During the completion of this exam, you may **NOT** check the official textbook, notes, or any material posted on the course website.
- The use of any on-line/physical/personal resources will be considered cheating.
- Each step on each answer must be justified. Indicate the methods you use to solve the equations (separable, guess and check, ...) to integrate function (u-substitution, by parts, partial fractions).
- By submitting the answers you agree to abide by the BU honor code. As instructors, we reserve our right, after the completion of the exam, to **ask privately** about your solutions to any of the problems.

1. (20 points) You are given the following linear system:

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 5 & -1 \\ 1 & 7 \end{pmatrix} \mathbf{Y}.$$

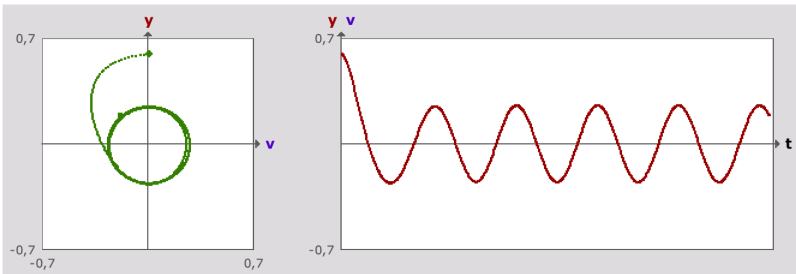
   (a) (15 points) Find the general solution and sketch the phase portrait. Make sure the eigenvalues are correct!!!

   (b) (5 points) Come up with a real life situation which can be represented by the model above.

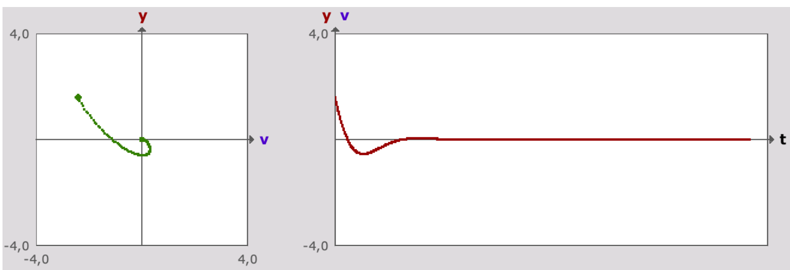
2. (20 points) Below are the plots of solutions to four different harmonic oscillators of equation

$$y'' + py' + qy = A\cos(\omega t),$$

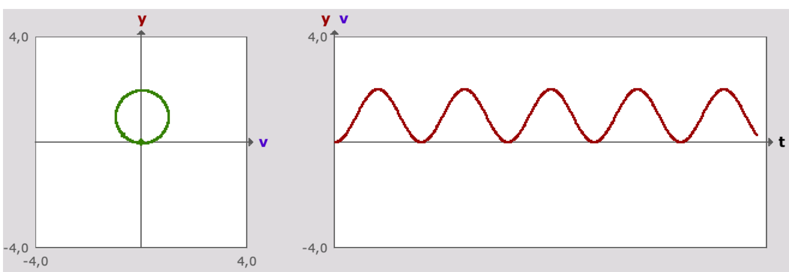
corresponding to different values of p, q, A and ω. Describe the type of oscillators by indicating if there is damping and/or forcing and what type. Indicate **possible** values of p, q, A and ω in each case. Indicate $A = 0$, if you think there is no forcing.



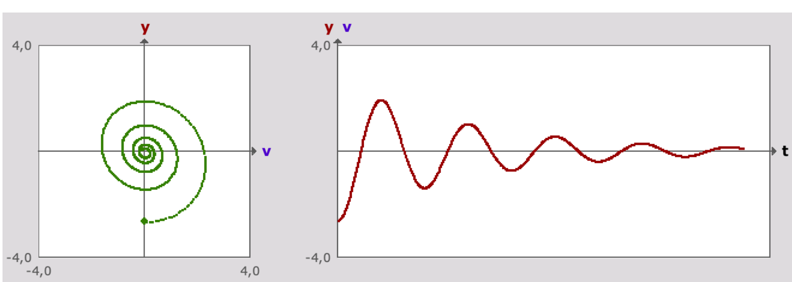
(i)



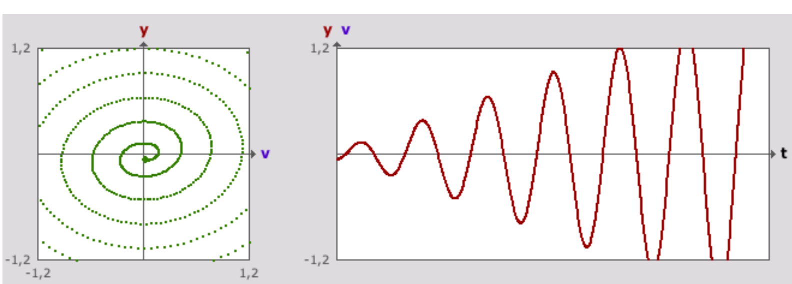
(ii)



(iii)



(iv)



(v)

3. (20 points) Consider the one-parameter family of linear systems, where $0 \leq \theta \leq 2\pi$, given as follows.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} \dfrac{\theta}{2} & -\dfrac{\sin(\theta)}{2} \\ \dfrac{\theta^2}{2} & \dfrac{\theta}{2} \end{pmatrix} \mathbf{Y}$$

(a) (10 points) Compute the trace and determinant and sketch the curve as θ varies in the trace-determinant plane. Make sure to add any relevant curves on your sketch.

(b) (10 points) Describe the bifurcations that occur in the parameter θ.

(**Hint:** Compare the curve with the parabola that corresponds to linear systems with repeated eigenvalues.)

4. (20 points) Let BUID(n) be the n-th **nonzero** number of your BUID. Consider a harmonic oscillator with mass $m = \text{BUID}(1)$, spring constant $k = \text{BUID}(2)$, and damping coefficient $b = \text{BUID}(3)$.

   (a) (0 points) Write your BUID (in case you are not a BU student, use your date of birth MMDDYYYY).

   (b) (5 points) Write down the second-order differential equation, which models this oscillator, as well as the corresponding first-order system.

   (c) (5 points) Classify the oscillator and, when appropriate, give the natural period. Describe how typical solutions behave.

   (d) (10 points) Find the eigenvalues and eigenvectors of the linear system, as well as the general solution.