MIDTERM EXAM I

Last name: Name:

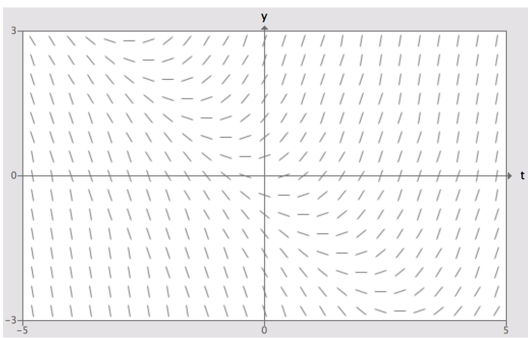
BUID:

Please do all of your work in this exam booklet and make sure that you cross any work that we should ignore when we grade. Books and extra papers are not permitted. If you have a question about a problem, pease ask. Remember: answers that are written logically and clearly will receive higher scores. Calculators are not allowed.

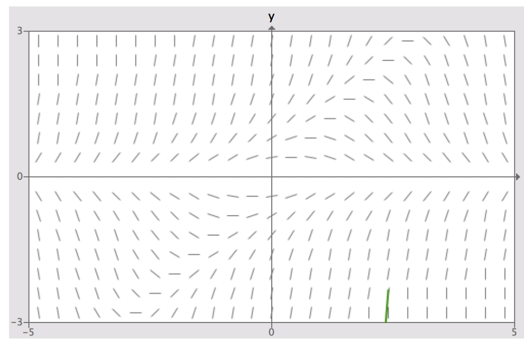
Problem n^o	**Possible points**	**Score**
1	10	
2	14	
3	16	
4	15	
5	10	
6	20	
7	15	
Total:	100	

Problem 1. Indicate which slope field corresponds to each of the following differential equations:

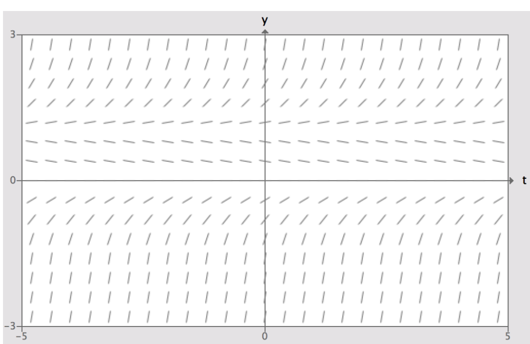
- $y' = y(y-1)$
- $y' = \sin(t) + t^2$
- $y' = y(y-t)$
- $y' = y + t$



(a)



(b)



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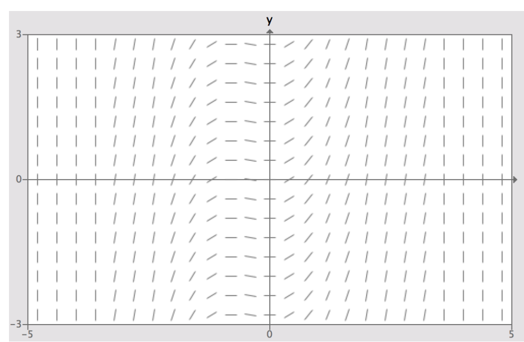


(d)

In each of the graphs above, sketch the solution with initial condition $y(0) = 1$.

Problem 2. Answer **TRUE** or **FALSE** to each of the following statements and justify your answer in **ONE** sentence.

- To find a particular solution of the differential equation $y' = 3y + e^{3t}$, the correct guess is $y_p(t) = ae^t$.

- The differential equation $y' = y^2 t + e^{2t}$ is linear.

- The differential equation $y' = 2ye^{4t} + y\sin(t)$ is separable.

- There is a unique solution to $y' = y^2 + 1$ with initial condition $y(-2) = 10$.

- The one-parameter family of differential equations $y' = y^4 + \mu$ has a bifurcation at $\mu = 0$.

- The fact that $y(t) = 0$ and $y(t) = t^3$ are both solutions to $y' = 3y^{2/3}$ contradicts the uniqueness theorem.

Problem 3. Consider the autonomous differential equation

$$\frac{dy}{dt} = y(y-1)(y-4).$$

- Draw its phase line and **classify** the equilibrium points.

- Without explicitly solving the equation, say all you can about the long-term behaviour of the solutions. Do it for solutions with different initial values.

- To solve this equation using separation of variables, partial fractions are required. Since you do **NOT** remember this method, you decide to use Euler's method and give an approximation instead. Starting from the initial condition $y(0) = 3$, calculate the first 2 of the method. Take a step size of $\Delta t = 0.5$.

n	t_n	y_n	$f(t_n, y_n)$
0			
1			
2			

- What went wrong? Compare the result to your answer in the second question of this problem.

Problem 4. The interaction between a population of sharks and crustaceans can be modelled via the following predator-pray system.

$$\begin{cases} \frac{dx}{dt} = 5x - 100xy \\ \frac{dy}{dt} = -2y + 0.5xy \end{cases}$$

- Indicate which variable x or y corresponds to each of the species.

- Find the equilibria of this system.

- There is a huge scale difference between the coefficients of the interaction terms. Give a real-life explanation of this.

- Recent studies have estimated that, due to shark finning (removal of shark fins while alive), the rate of death of sharks has increased. Modify one or more parameters in the system to account for this illegal practice. Find the new equilibria.

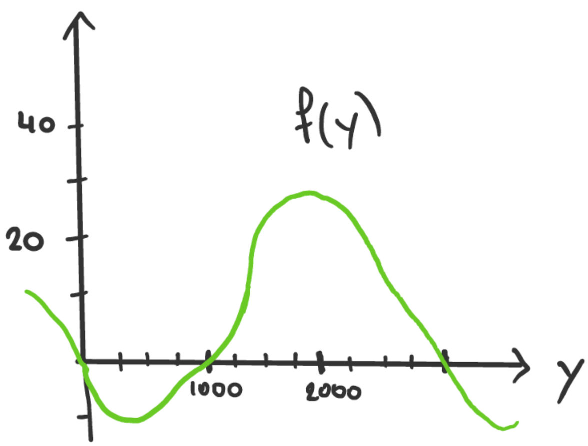
Problem 5. Find the solution of the following initial value problem

$$\frac{dy}{dt} = (y+1)\left(\cos(2t) + \frac{1}{t}\right), \qquad y(\pi) = 1.$$

Problem 6. A population of rhinoceros $y(t)$ can be modelled with an autonomous differential equation

$$\frac{dy}{dt} = f(y),$$

where the graph of $f(y)$ is given. The values on the graph are **approximate**, no calculations required.



- Draw the phase line associated to this equation.

- If the population of rhinos is $y(0) = 500$, what happens in the long term? And what if $y(0) = 1500$? Give a real-life explanation for that.

- To save the rhinos, some NGOs perform certain actions. This actions can be modelled by adding a parameter μ in the equation.

$$\frac{dy}{dt} = f(y) + \mu.$$

What is the minimum value of μ, so that no matter what the initial population of rhinos is, they will never become extinct? (Yes, this is a bifurcation problem)

- Draw the bifurcation diagram.

Problem 7. Solve the following initial value problem:

$$y' + \frac{y}{t} = t^2 - 1 + \frac{3}{t} \qquad y(1) = 0.$$

BONUS

As mentioned earlier in class, an system consisting of an object attached to a spring can be modeled by the second order differential equation

$$y'' = -4y.$$

Use your calculus knowledge to find the general solution to this equation. Solve for the initial values $y(0) = 4$ and $y'(0) = 1$.