RLC Circuits

NOTE: This worksheet is LAB 3.2 in the textbook.

We have already seen examples of differential equations that serve as models of simple electrical circuits involving only a resistor, a capacitor, and a voltage source. In this lab we consider slightly more complicated circuits consisting of a resistor, a capacitor, an inductor, and a voltage source (see Figure 3.67). The behavior of the system can be described by specifying the current moving around the circuit and the changes in voltages across each component of the circuit. In this lab we take an axiomatic approach to the relationship between the current and the voltages. Readers interested in more information on the derivation of these laws are referred to texts in electric circuit theory.

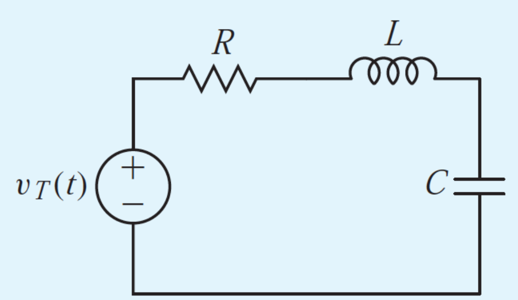

Figure 3.67
An RLC circuit.

Following the conventions used by electrical engineers, we let i denote the current moving around the circuit. We let v_T, v_C, and v_L denote the voltages across the voltage source, the capacitor, and the inductor, respectively. Also, we let R denote the resistance, C the capacitance, and L the inductance of the associated components of the circuit (see Figure 3.67). We think of v_T, R, C, and L as parameters set by the person building the circuit. The quantities i, v_C, and v_L depend on time. We need the following basic relationships between the quantities above. First, Kirchhoff's voltage law states that the sum of the voltage changes around a closed loop must be zero. For our circuit this gives

$$v_T - Ri = v_C + v_L.$$

Next, we need the relationship between current and voltage in the capacitor and the inductor. In a capacitor the current is proportional to the rate of change of the voltage. The proportionality constant is the capacitance C. Hence we have

$$C\frac{dv_C}{dt} = i.$$

In an inductor, the voltage is proportional to the rate of change of the current. The proportionality constant is the inductance L. Hence we have

$$L\frac{di}{dt} = v_L.$$

In this lab we consider the possible behavior of the circuit above for several different input voltages. In your report, address the following questions:

- First, set the input voltage to zero, that is, assume $v_T = 0$. Using the three equations above, write a first-order system of differential equations with dependent variables i and v_C. [Hint: Use the first equation to eliminate v_L from the third equation. You should have R, C, and L as parameters in your system.]

- Find the eigenvalues of the resulting system in terms of the parameters R, C, and L. What are the possible phase planes for your system given that R, C, and L are always nonnegative? Sketch the phase plane and the $v_C(t)$- and $i(t)$-graphs for each case.
- Convert the first-order system of equations from Part 1 into a second-order differential equation involving only v_C (and not i). (This is the form of the equation that you will typically find in electric circuit theory texts.)
- Repeat Part 1, assuming that v_T is nonzero. The resulting system will have R, C, L, and v_T as parameters.
- The units used in applications are volts and amps for voltages and currents, ohms for resistors, farads for capacitors, and henrys for inductors. A typical, off-the-shelf circuit might have parameter values $R = 2000$ ohms (or 2 kilo-ohms), $C = 2 \cdot 10^{-7}$ farads (or 0.2 microfarads), and $L = 1.5$ henrys. Assuming zero input, $v_T = 0$, and that the initial values of the current and voltage are $i(0) = 0$ and $v_C(0) = 10$, describe the behavior of the current and voltage for this circuit.
- Repeat Part 5 using a voltage source of $v_T = 10$ volts.